

TRANSLATIONS OF FUNCTIONS

Vertical stretch of 5

Shrink is a fraction

Shift right 3 units

(when you take it out the

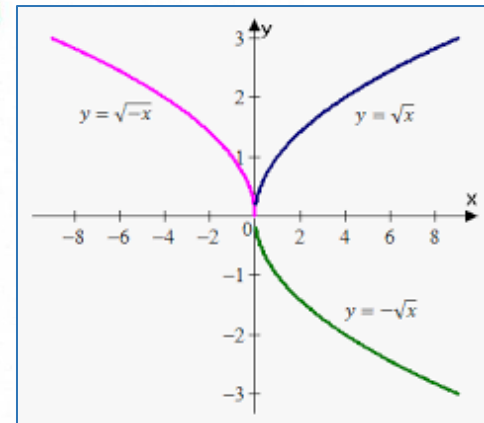
parenthesis it changes the sign)

$$f(x) = -5(x-3)^2 + 7$$

Negative in front
reflects across the x-axis

Shifts up 7 units

REFLECTION:



$f(x) = \sqrt{-x}$ means it reflects across the y-axis

Horizontal Stretch is fraction inside the function $f(x) = (\frac{1}{5}x)^2$

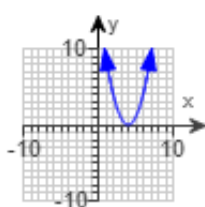
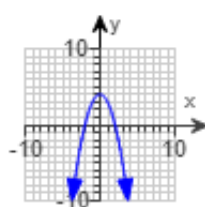
Horizontal Shrink is whole # inside the function $f(x) = (3x)^2$

- 1) Suppose that the graph of a function f is known. Then the graph of $y = f(x - 2)$ may be obtained by a horizontal shift of the graph of f to the right a distance of 2 units.
- 2) Suppose that the graph of a function f is known. Then the graph of $y = f(-x)$ may be obtained by a reflection about the y-axis of the graph of the function $y = f(x)$.
- 3) Which of the following functions has a graph that is the graph of $y = \sqrt{x}$ shifted up 5 units?

Choose the correct answer below.

- ☐ $y = \sqrt{x+5}$
- ☐ $y = \sqrt{x-5}$
- ☒ $y = \sqrt{x} + 5$
- ☐ $y = \sqrt{x} - 5$

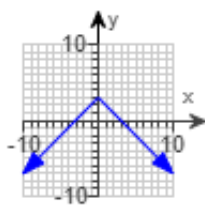
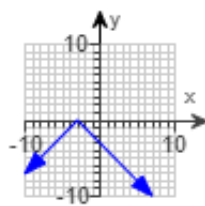
4) Drag the function given above into the appropriate area below to match the graph.



Points down (- in front)

$$y = -x^2 + 4$$

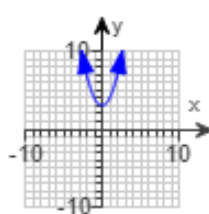
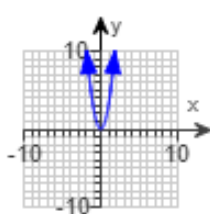
$$y = (x - 4)^2$$



$$y = -|x + 3|$$

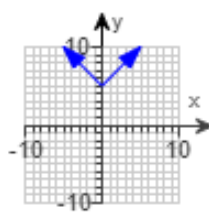
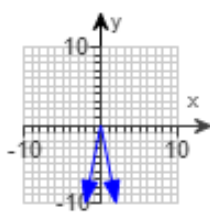
$$y = -|x| + 3$$

5) Drag the function given above into the appropriate area below to match the graph.



$$y = 3x^2$$

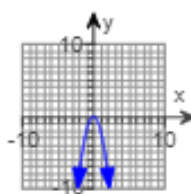
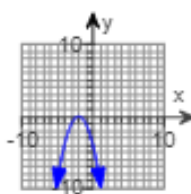
$$y = x^2 + 3$$



$$y = -5|x|$$

$$y = |x| + 5$$

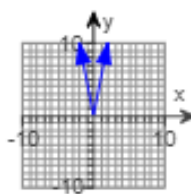
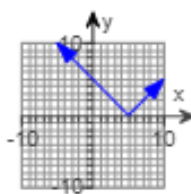
6) Drag the function given above into the appropriate area below to match the graph.



Points down (- in front)

$$y = -(x+2)^2$$

$$y = -2x^2$$



$$y = |x-5|$$

$$y = 5|x|$$

7) Write the function whose graph is the graph of $y = x^3$, but is shifted to the left 8 units.
 $y = (x+8)^3$ left and right (inside)

8) Write the function whose graph is the graph of $y = |x|$, but is shifted up 1 unit.
 $y = |x|+1$ up and down (outside)

9) Write the function whose graph is the graph of $y = 2\sqrt{x}$ but is reflected about the y-axis.

The function is $y = 2\sqrt{-x}$.

Negative inside

(Type an exact answer, using radicals as needed.)

10) Find the function that is finally graphed after the following transformations are applied to the graph of $y = \sqrt{x}$ in the order listed.

- (1) Reflect about the x-axis
- (2) Shift down 4 units
- (3) Shift right 8 units

Negative in front
 up and down (outside)
 up and down (outside)

$$y = -\sqrt{x-8} - 4$$

11) Find the function that is finally graphed after the following transformations are applied to the graph of $y = \sqrt{x}$ in the order listed.

- (1) Vertical stretch by a factor of 3
- (2) Shift up 1 unit
- (3) Shift left 5 units

$$y = 3\sqrt{x+5} + 1$$

12) Graph the following function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function $y = x^2$ and show all stages. Be sure to identify at least three key points. Find the domain and the range of the function.

$$f(x) = x^2 - 11$$

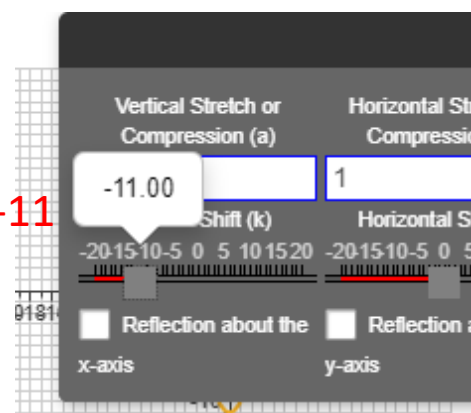
Which transformations are needed to graph the function $f(x) = x^2 - 11$? Choose the correct answer below.

- ☒ A. The graph of $y = x^2$ should be shifted vertically down by 11 units.

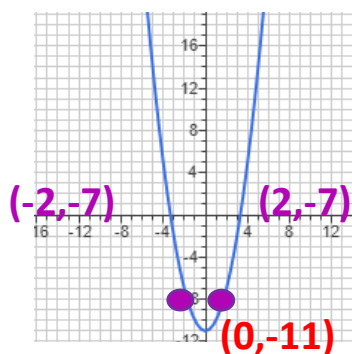
Click parabola



Click anywhere on the graph



Vertical shift of -11



Points that lie on the graph of $y = x^2$ (Simplify your answers.)	Corresponding points that lie on the graph of $f(x) = x^2 - 11$ (Type ordered pairs. Simplify your answer.)
$(-2, 4)$	$(-2, -7)$
$(0, 0)$	$(0, -11)$
$(2, 4)$	$(2, -7)$

First column

$$\begin{aligned} (-2)^2 &= 4 \\ (0)^2 &= 0 \\ (2)^2 &= 4 \end{aligned}$$

$$y = (-2)^2 - 11 = -7$$

TWO units to the left of vertex for x

VERTEX

TWO units to the right of vertex for x

$$y = (2)^2 - 11 = -7$$

Two units left then right from this 2

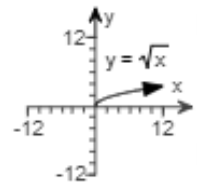
The domain of $f(x)$ is $(-\infty, \infty)$. (x) move your pencil from left to right on the graph
(Type your answer in interval notation.)

The range of $f(x)$ is $[-11, \infty)$. (y) move your pencil from bottom to top on the graph
(Type your answer in interval notation.)

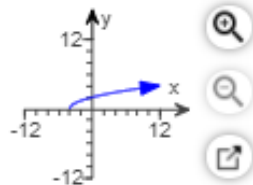
13)

Graph the following function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function shown to the right. Find the domain and range of the function.

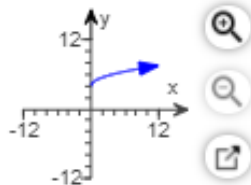
$$h(x) = \sqrt{x-4}$$



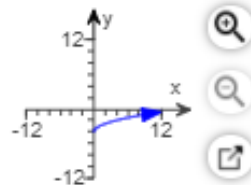
☐ A.



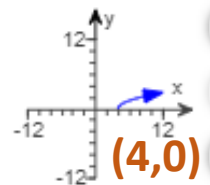
☐ B.



☐ C.



☒ D.



Find the domain of $h(x)$.

$[4, \infty)$ (x)
(Type your answer in interval notation.)

both always to ∞ on $y = \sqrt{x}$

Find the range of $h(x)$.

$[0, \infty)$ (y)
(Type your answer in interval notation.)

- 14) Graph the following function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function $y = x^3$ and show all stages. Be sure to identify at least three key points. Find the domain and the range of the function

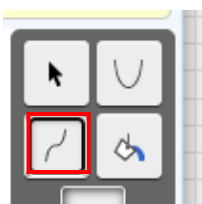
$$f(x) = (x + 2)^3 - 1 \quad \text{vertex } (-2, -1)$$

Which transformations are needed to graph the function $f(x) = (x + 2)^3 - 1$? Choose the correct answer below.

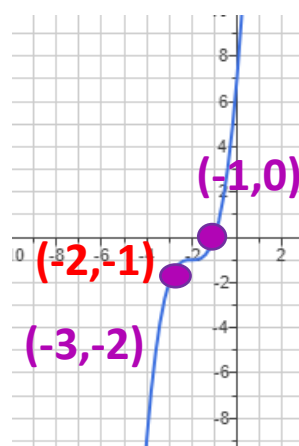
- ☒ A. The graph of $y = x^3$ should be horizontally shifted to the left by 2 units and shifted vertically down by 1 unit.

Graph is cube root

Click anywhere on the graph



Vertical Stretch or Compression (a)	Horizontal Stretch or Compression (a)
<input type="text" value="1"/>	<input type="text" value="1"/>
Vertical Shift (k)	Horizontal Shift (h)
<input type="text" value="-1"/>	<input type="text" value="-2"/>
<input type="checkbox"/> Reflect over x-axis	<input type="checkbox"/> Reflect over y-axis



Points that lie on the graph of $y = x^3$ (Simplify your answers.)	Corresponding points that lie on the graph of $f(x) = (x + 2)^3 - 1$ (Type ordered pairs. Simplify your answer.)	
$(-1, -1)$	$(-3, -2)$	ONE units to the left of vertex for x
$(0, 0)$	$(-2, -1)$	VERTEX
$(1, 1)$	$(-1, 0)$	ONE units to the right of vertex for x

First column

$(-1)^3 = -1$

$(0)^3 = 0$

$(1)^3 = 1$

$y = (-3+2)^3 - 1 = -2$

$y = (-1+2)^3 - 1 = 0$

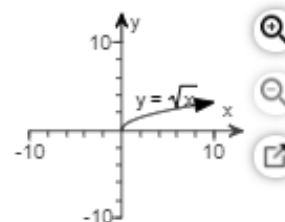
ONE units left then right from this 1

No restrictions on either: The domain of $f(x)$ is $(-\infty, \infty)$.
(Type your answer in interval notation.)

The range of $f(x)$ is $(-\infty, \infty)$.
(Type your answer in interval notation.)

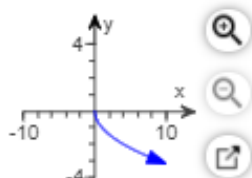
- 15) Graph the following function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function shown to the right. Find the domain and range of the function.

$$f(x) = -\sqrt{x}$$

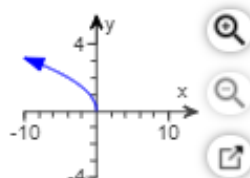


Choose the correct graph below.

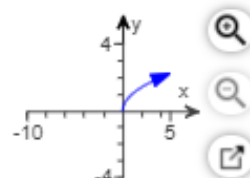
☒ A.



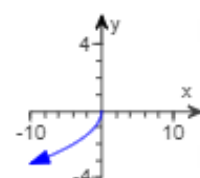
☐ B.



☐ C.



☐ D.



The domain of $y = -\sqrt{x}$ is $[0, \infty)$.

(Type your answer in interval notation.)

The range of $y = -\sqrt{x}$ is $(-\infty, 0]$.

(Type your answer in interval notation.)

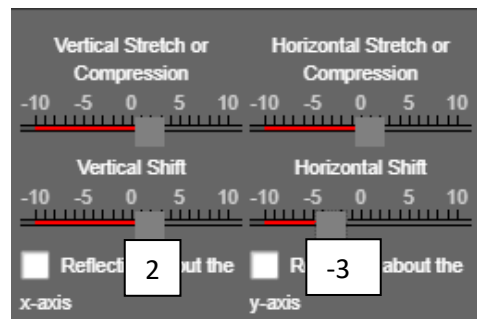
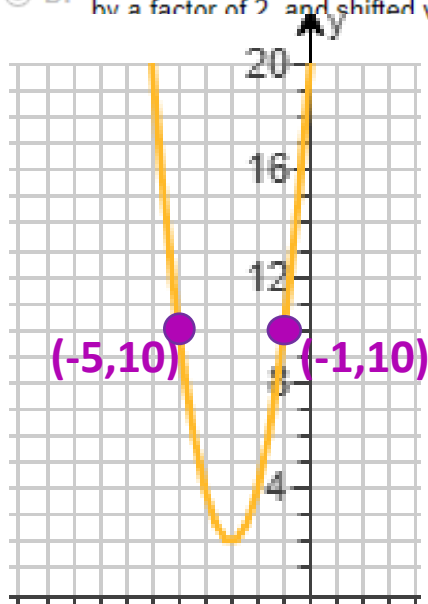
- 16) Graph the following function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function $y = x^2$ and show all stages. Be sure to identify at least three key points. Find the domain and the range of the function.

VERTEX (-3,2)

$$f(x) = 2(x+3)^2 + 2$$

☒ D.

The graph of $y = x^2$ should be horizontally shifted to the left by 3 units, vertically stretched by a factor of 2, and shifted vertically up by 2.



Points that lie on the graph of $y = x^2$ (Simplify your answers.)	Corresponding points that lie on the graph of $f(x) = 2(x + 3)^2 + 2$ (Type ordered pairs. Simplify your answer.)	
$(-2, 4)$	$(-5, 10)$	TWO units to the left of vertex for x
$(0, 0)$	$(-3, 2)$	VERTEX
$(2, 4)$	$(-1, 10)$	TWO units to the right of vertex for x

First column

$(-2)^2 = 4$
 $(0)^2 = 0$
 $(2)^2 = 4$

$y = 2(-5+3)^2 + 2 = 10$

$y = 2(-1+3)^2 + 2 = 10$

Two units left then right from this 2

The domain of $f(x)$ is $(-\infty, \infty)$.
(Type your answer in interval notation.)

The range of $f(x)$ is $[2, \infty)$.
(Type your answer in interval notation.)

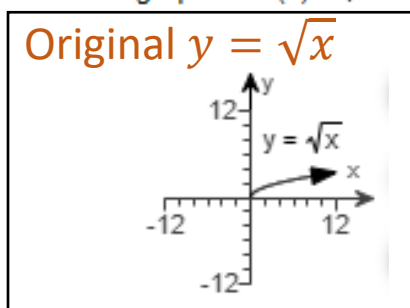
- 17) Graph the following function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function $y = \sqrt{x}$ and show all stages. Be sure to show at least three key points. Find the domain and the range of the function.

$$h(x) = \sqrt{-x} - 6$$

Which transformations are needed to graph the function $h(x) = \sqrt{-x} - 6$? Choose the correct answer below.

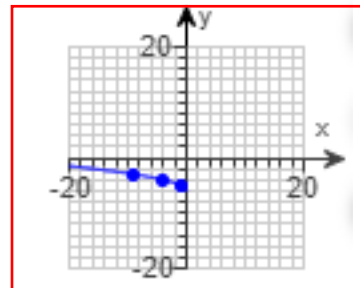
- ☒ C. The graph of $y = \sqrt{x}$ should be vertically shifted down by 6 units, reflected about the y-axis.

Choose the correct graph for $h(x) = \sqrt{-x} - 6$ below.



VERTEX $(0, -6)$

shifts down 6 and reflects across the y-axis



The domain of $h(x)$ is $(-\infty, 0]$. X VALUE left to right, ∞ to 0 including 0
(Type your answer in interval notation.)

The range of $h(x)$ is $[-6, \infty)$. Y VALUE bottom to top, includes -6 to ∞
(Type your answer in interval notation.)

- 18) Starting with the graph of a basic function, graph the following function using the techniques of shifting, compressing, stretching, and/or reflecting. Find the domain and range of the function.

$$g(x) = 5(x + 3)^2 - 2$$

Use the graphing tool to graph the equation.

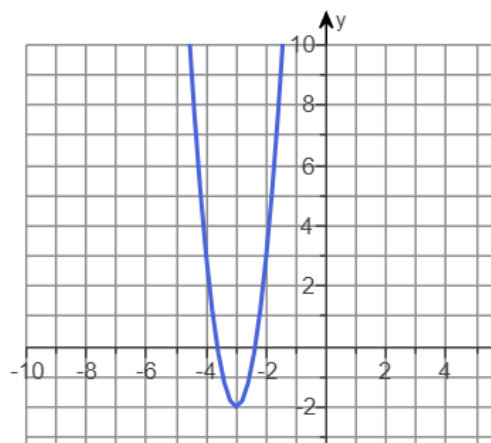


The domain of $g(x)$ is $(-\infty, \infty)$.

(Type your answer in interval notation.)

The range of $g(x)$ is $[-2, \infty)$.

(Type your answer in interval notation.)



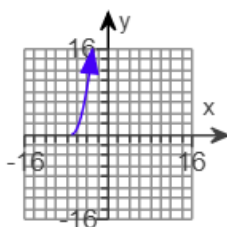
Vertical Stretch or Compression (a)	Horizontal Stretch or Compression (a)
<input type="text" value="5"/>	<input type="text" value="1"/>
Vertical Shift (k)	Horizontal Shift (h)
<input type="text" value="-2"/>	<input type="text" value="-3"/>
<input type="checkbox"/> Reflect over x-axis	<input type="checkbox"/> Reflect over y-axis

- 19) Graph the relation. Determine the domain and range, and whether the relation is a function.

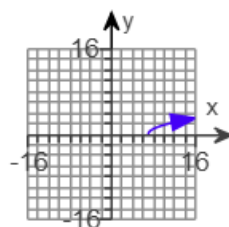
$$y = \sqrt{x} - 7$$

Down 7

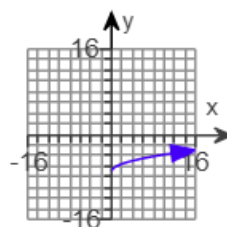
☐ A.



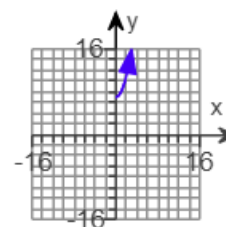
☐ B.



☒ C.



☐ D.



The domain of the relation is $[0, \infty)$.

(Type your answer in interval notation.)

The range of the relation is $[-7, \infty)$.

(Type your answer in interval notation.)

Is y a function of x ?



Yes

20) Find the domain and range of the function.

$$f(x) = \sqrt{x-9} + 3$$

Right 9 (domain) up 3 (range)

The domain is $[9, \infty)$.

(Type your answer in interval notation.)

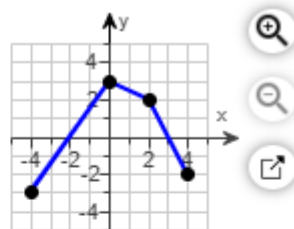
The range is $[3, \infty)$.

(Type your answer in interval notation.)

21) The graph of a function f is illustrated. Use the graph of f as the first step toward graphing each of the following functions.

(a) $Q(x) = f(x + 3)$

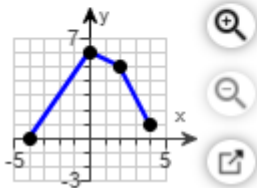
(b) $P(x) = -f(x)$



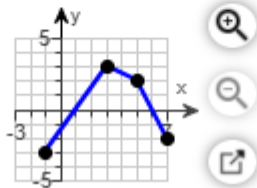
(a) Choose the graph of $Q(x) = f(x + 3)$ below.

left 3

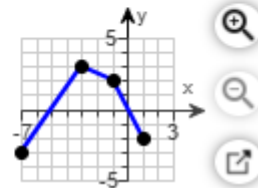
☐ A.



☐ B.



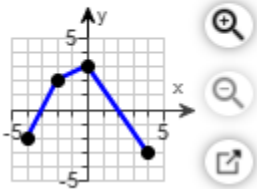
☒ C.



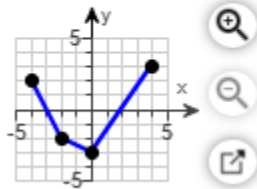
(b) Choose the graph of $P(x) = -f(x)$ below.

reflects across the x-axis

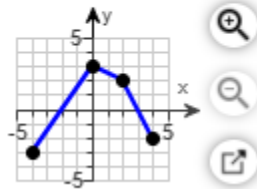
☐ A.



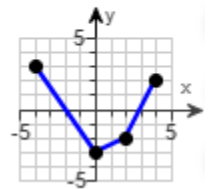
☐ B.



☐ C.



☒ D.

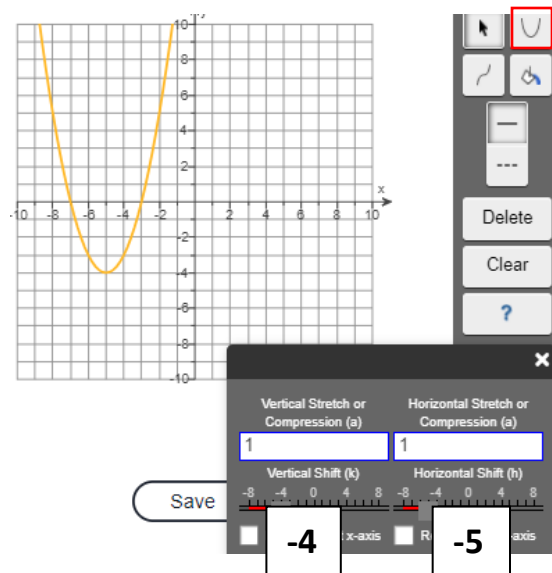


- 22) Graph the function f by starting with the graph of $y = x^2$ and using transformations (shifting, compressing, stretching, and/or reflecting). [Hint: If necessary, write f in the form $f(x) = a(x - h)^2 + k$.]

$$f(x) = (x + 5)^2 - 4$$

Which transformations are needed to graph the function $f(x) = (x + 5)^2 - 4$? Choose the correct answer below.

- ☐ A. The graph of $y = x^2$ should be horizontally shifted to the left by 4 units and shifted vertically up by 5 units.
- ☒ B. The graph of $y = x^2$ should be horizontally shifted to the left by 5 units and shifted vertically down by 4 units.



- 23) Graph the function f by starting with the graph of $y = x^2$ and using transformations (shifting, compressing, stretching, and/or reflecting). [Hint: If necessary, write f in the form $f(x) = a(x - h)^2 + k$.]

$$f(x) = x^2 + 2x - 3$$

Find x value of vertex using $x = \frac{-b}{2a} = \frac{-2}{2} = -1$

Which transformations are needed to graph the function $f(x) = x^2 + 2x - 3$? Choose the correct answer below.

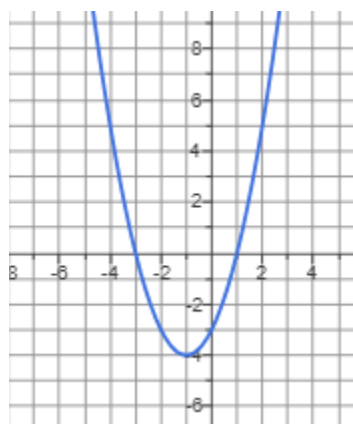
- ☐ A. The graph of $y = x^2$ should be horizontally shifted to the right by 1 unit and shifted vertically up by 4 units.
- ☒ B. The graph of $y = x^2$ should be horizontally shifted to the left by 1 unit and shifted vertically down by 4 units.

Find y by plugging -1 in for x
 $y = (-1)^2 + 2(-1) - 3 = -4$

Vertex is $(-1, -4)$

Moves left 1 and down 4 using vertex

Vertical shift is -4
 Horizontal shift is -1



- 24) Graph the function f by starting with the graph of $y = x^2$ and using transformations (shifting, compressing, stretching, and/or reflecting). [Hint: If necessary, write f in the form $f(x) = a(x - h)^2 + k$.]

$$f(x) = 2x^2 - 12x + 17$$

Find x value of vertex using $x = \frac{-b}{2a} = \frac{12}{4} = 3$

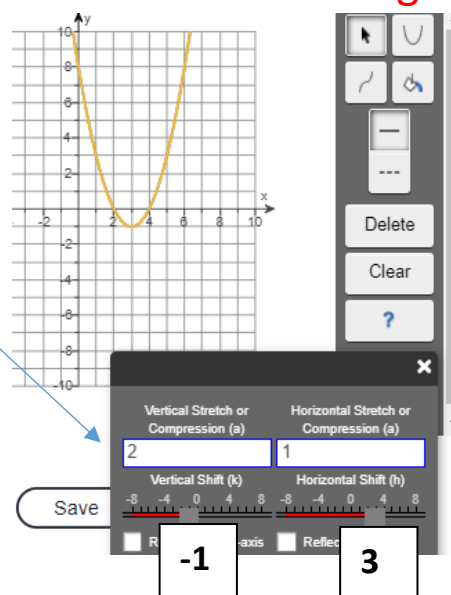
Which transformations are needed to graph the function $f(x) = 2x^2 - 12x + 17$? Choose the correct answer below.

- ☐ A. The graph of $y = x^2$ should be stretched vertically by a factor of 2, horizontally shifted to the left by 3 units, and shifted vertically up by 1 unit.
- ☒ B. The graph of $y = x^2$ should be stretched vertically by a factor of 2, horizontally shifted to the right by 3 units, and shifted vertically down by 1 unit.

Find y by plugging -1 in for x
 $y = 2(3)^2 - 12(3) + 17 = -1$

Vertex is (3,-1)

Moves right 3 and down 1 using vertex



- 25) Graph the function f by starting with the graph of $y = x^2$ and using transformations (shifting, compressing, stretching, and/or reflecting). [Hint: If necessary, write f in the form $f(x) = a(x - h)^2 + k$.]

$$f(x) = -x^2 - 2x$$

Find x value of vertex using $x = \frac{-b}{2a} = \frac{2}{-2} = -1$

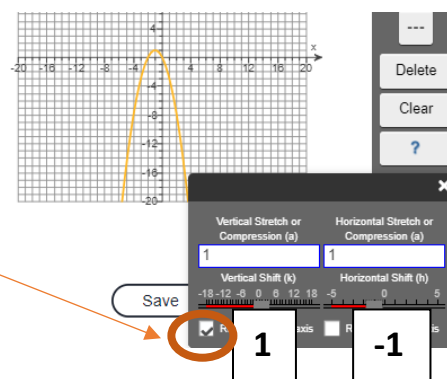
Which transformations are needed to graph the function $f(x) = -x^2 - 2x$? Choose the correct answer below.

- ☐ A. The graph of $y = x^2$ should be horizontally shifted to the right by 1 unit, reflected about the y -axis, and shifted vertically down 1 unit.
- ☒ B. The graph of $y = x^2$ should be horizontally shifted to the left by 1 unit, reflected about the x -axis, and shifted vertically up 1 unit.

Find y by plugging -1 in for x
 $y = -(-1)^2 - 2(-1) = 1$

Vertex is (-1,1)

Moves left 1 and up 1 using vertex



Make sure to check reflection FIRST

- 26) Graph the following function by starting with a function from the library of functions and then combining shifting, compressing, stretching, and/or reflecting techniques.

$$f(x) = 2(x + 7)^2 - 4$$

the given function using a function from the library of functions.

- ☐ A. Stretch the graph horizontally by a factor of 2.
- ☐ B. Reflect the graph about the y-axis.
- ☐ C. Shift the graph 7 units to the right.
- ☒ D. Stretch the graph vertically by a factor of 2.
- ☒ E. Shift the graph 7 units to the left.
- ☒ F. Shift the graph 4 units down.
- ☐ G. Reflect the graph about the x-axis.
- ☐ H. Shift the graph 4 units up.

